# ⬡ NEGATIVE NUMBERS (contributed by Torsten Reincke)

The problem

Negative numbers appear when subtracting a larger number from a smaller one, like in 23-61=-38. These negative numbers can not be displayed on the abacus directly since it is impossible to place e.g. minus 3 beads on a rod. The lack of a minus sign is circumvented by displaying negative numbers indirectly using the concept of complementary numbers. These are sometimes also called supplementary numbers.

Complementary numbers

For example, the complementary number of 3 with respect to 10 is 7. For 4 it is 6, for 8 it is 2. If you add two complementary numbers you always get a number that is a power of 10 (that is 10, 100, 1000, 10000 and so on). This means for example that the complementary number of 3 with respect to 100 is 997. So how do we use this on the abacus? If an arbitrary number is currently set on the abacus, you can immediately read the corresponding complementary number by looking at the beads that are not moved toward the beam but towards the upper and lower border of the frame:
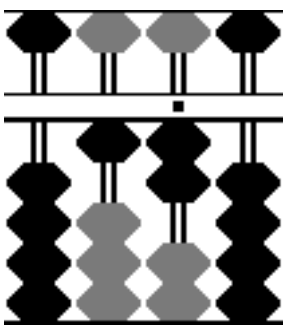
The number 12. The complement is 88 (represented by the grey beads).
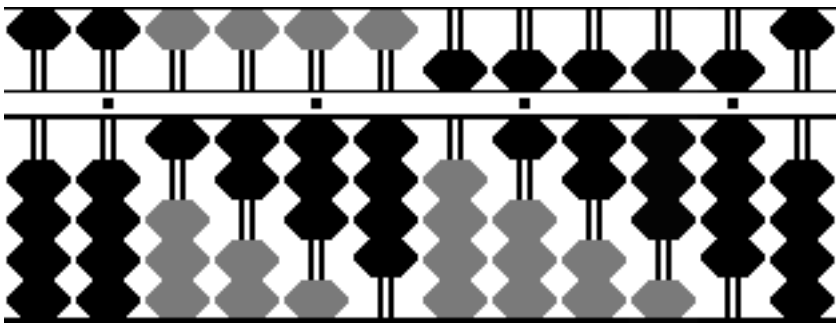
The number 123456789. The complement is 87654321 (again represented by the grey beads).

Note: *You have to add 1 to the last (i.e. rightmost) nonzero rod.*
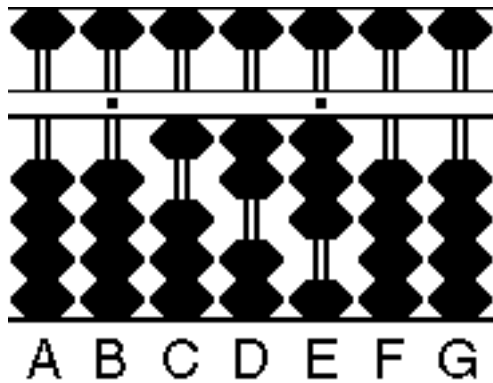
Subtraction using complementary numbers

Let us now solve the original problem, 23-61, using this concept. Place 23 on the abacus. Trying to subtract 61, we immediately see that this is impossible.
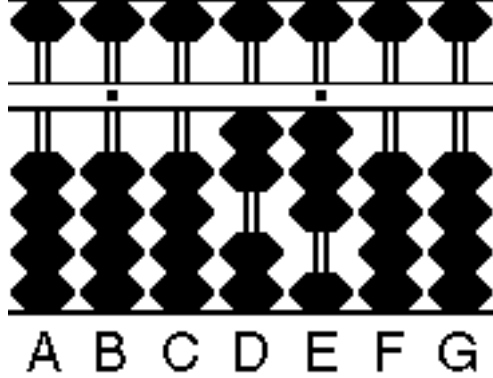
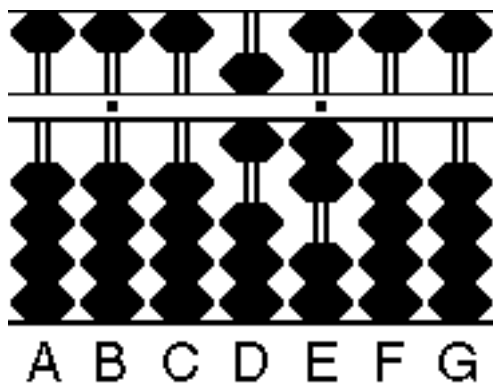The number 23. We can not subtract the 6 (from 61) on rod D.

[ABCDEFG -> 0002300].

Now we use a trick. We borrow 100 by adding 1 on rod C:

We borrowed 100 (i.e. added 1 on rod C).

Now the number is large enough to subtract 61. The result is shown below. The result of the problem is now the complementary number of 62 with respect to the 100 we borrowed a second ago. So the result is -38.


123 - 61 = 62. The result is the complement: -38.

We should save us one additional step in the example above. Instead of first adding 100 and then subtracting 60 (i.e. subtract 6 on rod D) we just add the complement of 6 (with respect to 10) on rod D instead and then subtract the 1 on rod D. Again, instead of adding 100 just add the complement on the rod where you need it. It is just like thinking: "Oh, I need to subtract 6 from 2. That's impossible. So I just add the complement of 6 here (that is 4) and remember that my result will become a negative number so I have to read the complementary number from the abacus, when I'm finished".
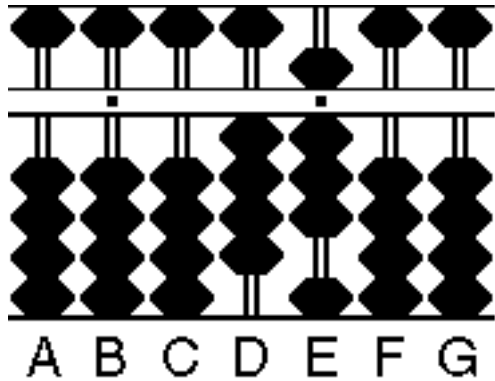
Golden rule 1: *When a digit is too large to be subtracted, add its complement instead.*

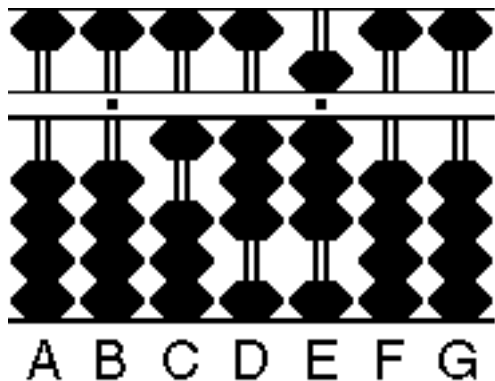**Golden rule 2:** *Using complements makes the calculation negative.*

Next example: 45-97+82
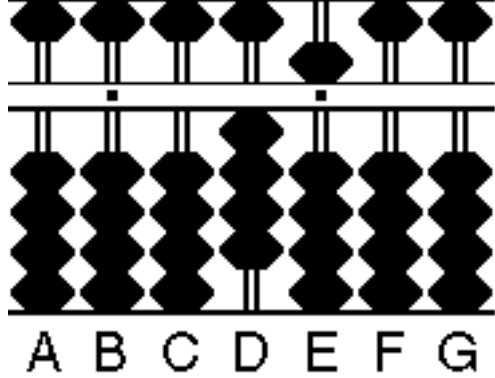
45.

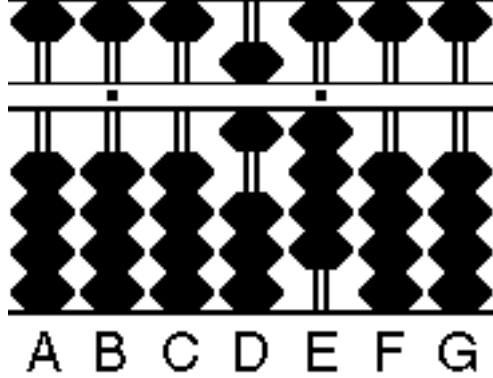We subtracted 97 (effectively borrowing 1 on rod C) This intermediate result is thus -52.

We added 82.

Here we have a number that is larger than the 100 we borrowed. So we can now safely subtract the 100 again and get a positive result that is 30. This is something we have to be aware of, when the calculation continues with other numbers that make the result positive again. So we have to remember on which rod we borrowed 1 (here rod C) and if we can subtract the 1 from the result we do it and get a positive number again. Note that the complement of 130 (=870) in the figure above would be the complement with respect to 1000 and we never borrowed 1000 but 100 and the result (870) is definitely wrong. (But of course, the complement of 130 with respect to our borrowed 100 is -30, and that is the correct result.)

Let's take another example: 47-83-3389
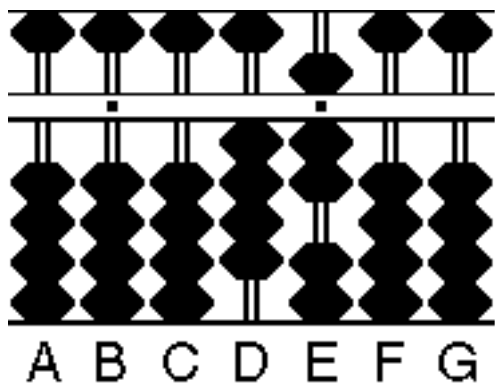
47.

We subtracted 83 (effectively borrowing 1 on rod C).

We have now already borrowed 100 but for the next step this is not enough. Ok, so we have to borrow again, but how much? Note here, that the complement on the abacus is always the complement to a power of 10 (10, 100, 1000, and so on). So if we now just borrowed another 10000 we would get a result which is the complement of 100+10000=10100. We cannot read this complement from the abacus. But if we borrow 9900 instead, the total amount borrowed will be 10000 and we are fine.
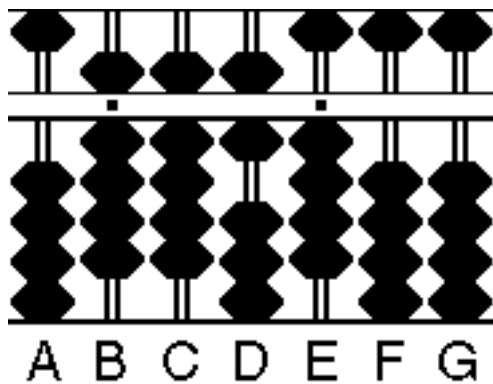
We borrowed another 9900.
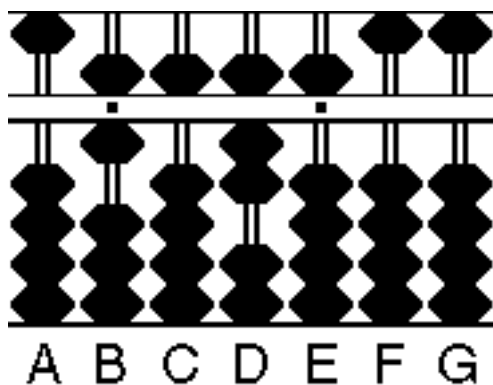
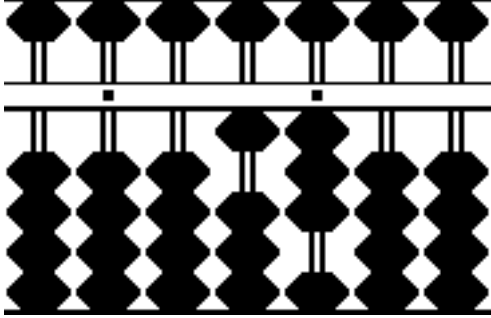We subtracted 3389. The result is the complement: -3425.

The process of borrowing additional amounts in the middle of a calculation is not difficult. You do not have to think 100+9900=10000. Just add 9es on the rods to the left of the current result until there is enough to subtract from and the significant 1 of the borrowed number will be one rod further left.

Golden rule 3: *Add 9es to the left when you need to borrow additional amounts.*

Next example: 12-99-1001
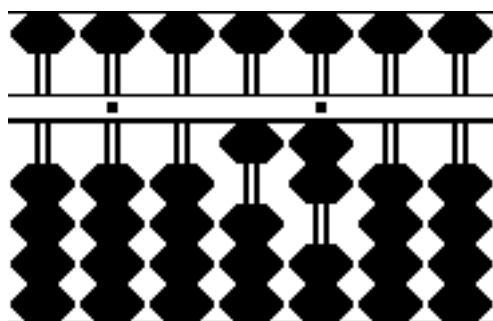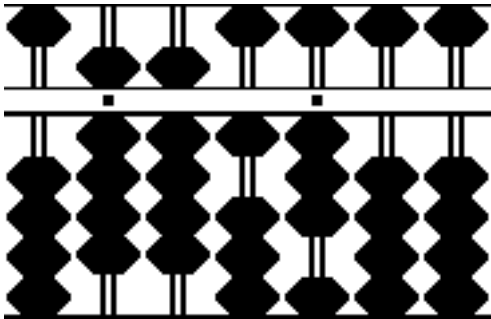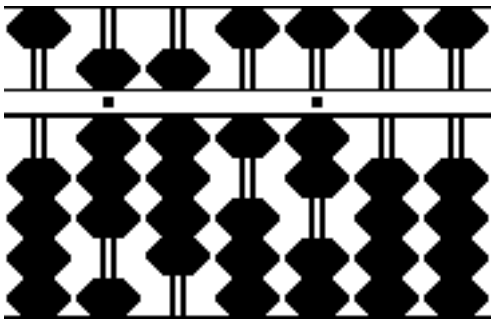
12.

We subtracted 99.

This looks like another shortcut. Instead of adding 100-99, just add 1.
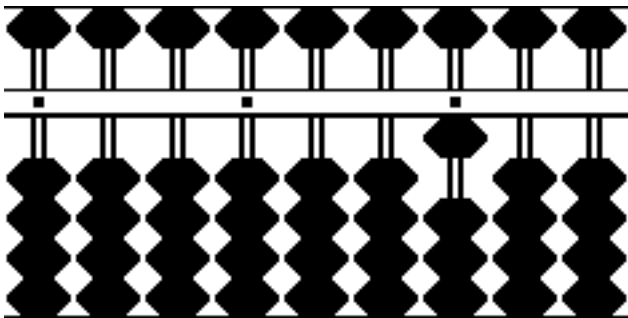

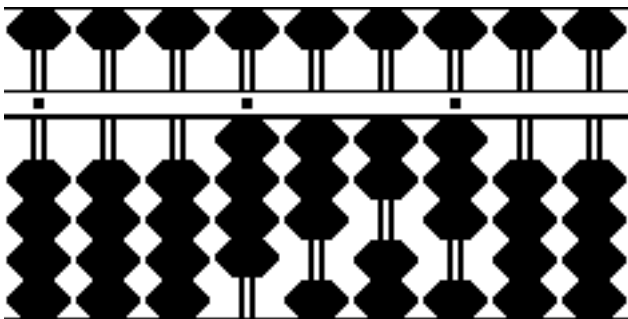We added additional 9es on the left until there is enough to subtract 1001.

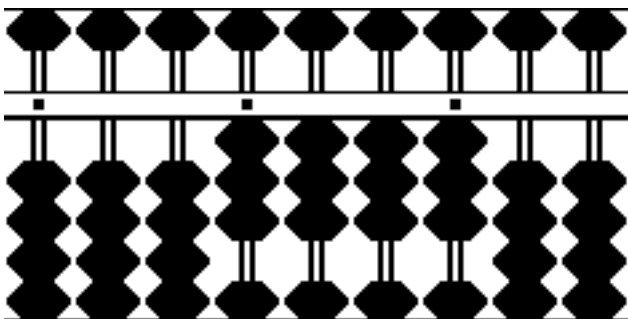We subtracted 1001 to get the total result -1088 as the complement.

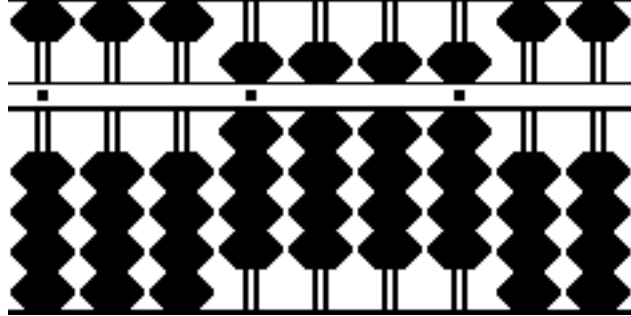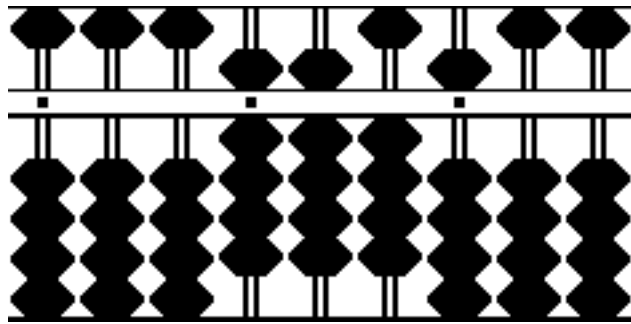Now a longer example: 1-5678-990+6666-54

1.

Subtracted 5678 (borrowing 10000).

Subtracted 990.

Added 6666.

Subtracted 54. The result is -55.

The concept of complementary numbers demonstrated here may sound familiar in knowing bead arithmetics. And in fact you will already have used it in normal addition and subtraction. Consider this: 17+18. First you add 1 on the ten's rod. Then, seeing that you can not add 8 to 7 on the one's rod, you add 1 on the ten's rod and subtract 3 on the one's rod. This is the equivalence of our rule number 1: if you can not add a digit, subtract the complement instead and add 1 on the next rod to the left.

Addition and subtraction also use another complement - the complement of 5. Try the example 2+4 yourself and see how the concept of complementary numbers is used here.

A Tomoe employee called these complements: "friends of 10" and "friends of 5". A nice term for an important concept.

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